

AIAA Journal

RUSSIAN SUPPLEMENT

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Published under National Science Foundation Grant-in-Aid. The Russian Supplement will appear monthly in 1963.

Radiant Heat Transfer in a Flowing Radiating Medium

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IN high temperature heat transfer installations (boiler units, industrial furnaces) radiant heat transfer between the flowing radiating combustion products of the fuel and the heated surfaces plays an important role. In Refs. 1-6, the problem of radiant heat transfer in a moving medium has been solved by assuming a uniform distribution of both the temperature and the velocity in a transverse cross section of the flow. The solution by Pukhov,⁵ which was obtained by assuming a uniform velocity distribution with respect to the cross section, does not contain a similar assumption with respect to the temperature. However, the radiation from the medium to the heated surface is considered only in the direction normal to the surface. In this paper an attempt is made to solve the problem, taking into consideration nonuniformity of velocities and temperatures in a transverse cross section of the flow of the radiating medium for cylindrical and plane channels, applicable to actual construction forms of heat transfer installations.

Statement of the Problem and General Solution

Through the channel of the given geometric configuration, there flows a "gray" radiating medium with a constant heat capacity and with a coefficient of attenuation k which consists, in the general case, of the coefficients of absorption α and

Translated from *Izvestiia Akademii Nauk SSSR, Otdel. Tekh. Nauk* (Bulletin of the Academy of Sciences USSR, Div. Tech. Sci.), no. 5, 46-50 (1958). Translated by U. S. Joint Publications Research Service, New York.

of scattering β ($k = \alpha + \beta$). Since this investigation deals only with the process of the radiant heat transfer in a moving medium, the medium is assumed to be not heat conducting ($\lambda = 0$).

The walls that limit the channel are "gray," and the distribution of emissivity and temperature along their surface is assumed given. The hydrodynamic properties of the flow are given by the distribution of velocity in the initial cross section of the channel, whereas the distribution of the streamlines of the medium is considered known. The initial temperature of the medium is taken as constant over the entire inlet cross section and equal to T_1 .

It is necessary to determine for the given conditions the regularity of the distribution of the temperatures in the flow of the medium and the heat transfer characteristics.

For the forementioned problem, for steady state conditions and the absence of sources of heat in the medium, the energy equation has the form

$$\operatorname{div} \mathbf{q}_r + \operatorname{div} \mathbf{q}_c = 0 \quad (1)$$

The divergence of the vector flux of convective heat transfer, with $c = \text{const}$, is

$$\operatorname{div} \mathbf{q}_c = (\gamma c \mathbf{w}, \operatorname{grad} T) \quad (2)$$

where γ , c , \mathbf{w} are, respectively, the density, heat capacity, and flow velocity of the medium at the point of volume under consideration; T ($^{\circ}\text{K}$) is the absolute temperature of the medium at the same point.

The divergence of the vector flux of radiant energy transfer

at the point of the volume under consideration is a function of the temperature field in the entire volume of the medium V and on the boundary surface F .

It can be represented in integral form as follows:

$$\operatorname{div} \mathbf{q}_\pi(M) = \left\{ \eta_c(M) - \alpha(M) \int_{(V)} \Gamma_{V-V}(M, P) \eta_c(P) dV_P - \alpha(M) \int_{(F)} \Gamma_{F-V}(M, S) E_c(S) dF_S \right\} \quad (3)$$

Here, $\eta_c = 4\alpha\sigma_0 T^4$ is the emission per unit volume of the medium at the indicated points of the volume (M is considered a fixed point, and P a variable point in the medium); $E_c(S) = A(S)\sigma_0 T^4(S)$ is the emission per unit area at the variable point S on the surface F which bounds the volume of the medium V ; further, $\Gamma_{V-V}(M, P)$ and $\Gamma_{F-V}(M, S)$ are the resolvents of the kernels:

$$K_{V-V}(M, P) = \frac{e^{-L_{MP}}}{4\pi l_{MP}^2}$$

$$K_{F-V}(M, S) = \frac{e^{-L_{MS}} \cos \theta_S}{\pi l_{MS}^2} \quad (L_{MP} = kl_{MP})$$

which are determined by the converging series of the type

$$\Gamma_{V-V}(M, P) = K_{V-V}(M, P) + \sum_{s=1}^{\infty} K_{V-V_s}(M, P)$$

$$\Gamma_{F-V}(M, S) = K_{F-V}(M, S) + \sum_{s=1}^{\infty} K_{F-V_s}(M, S)$$

Here, $K_{V-V_s}(M, P)$ and $K_{F-V_s}(M, S)$ represent the iterations of the s order of the kernels $K_{V-V}(M, P)$ and $K_{F-V}(M, S)$.

For the case in which the medium is purely absorbing, which we will consider to be the case henceforth, the scattering coefficient is equal to zero ($\beta \equiv 0$) and thus $k = \alpha$, and Eq. (3) becomes

$$\operatorname{div} \mathbf{q}_\pi(M) = 4k(M)\sigma_0 T^4(M) - k(M) \times$$

$$\int_{(V)} \Gamma_{V-V}(M, P) 4k(P)\sigma_0 T^4(P) dV_P - k(M) \times$$

$$\int_{(F)} \Gamma_{F-V}(M, S) A(S)\sigma_0 T^4(S) dF_S \quad (4)$$

From an examination of the thermodynamic equilibrium of the system, and on the basis of the second law of thermodynamics, we have the condition for a closed system:

$$\int_{(V)} \Gamma_{V-V}(M, P) k(P) dV_P +$$

$$\int_{(F)} \frac{\Gamma_{F-V}(M, S)}{4} A(S) dF_S = 1 \quad (5)$$

Using Eq. (5), Eq. (4) can be reduced to the form:

$$\operatorname{div} \mathbf{q}_\pi(M) = 4k(M)\sigma_0 \int_{(V)} [T^4(M) - T^4(P)] \Gamma_{V-V}(M, P) k \times$$

$$(P) dV_P + 4k(M)\sigma_0 \int_{(F)} [T^4(M) - T^4(S)] \frac{\Gamma_{F-V}(M, S)}{4} \times$$

$$A(S) dF_S \quad (6)$$

Disregarding further the radiant heat transfer between the individual volume elements of the medium, Eq. (6) can be written in the form:

$$\operatorname{div} \mathbf{q}_\pi(M) = 4k(M)\sigma_0 \times$$

$$\int_{(F)} [T^4(M) - T^4(S)] \frac{\Gamma_{F-V}(M, S)}{4} A(S) dF_S \quad (7)$$

Equation (7) will be used to determine the temperature field in the flow of the radiating medium.

Taking the temperature of the walls of the channel to be constant over the entire surface F and equal to $T(S) = T_{\text{wall}} = \text{const}$, Eq. (7) becomes:

$$\operatorname{div} \mathbf{q}_\pi(M) = 4k(M)\sigma_0 [T^4(M) - T_{\text{wall}}^4] f_d(M, N_{Bu}, A_{\text{wall}}) \quad (8)$$

$$f_d(M, N_{Bu}, A_{\text{wall}}) = \int_{(F)} \frac{\Gamma_{F-V}(M, S)}{4} A(S) dF_S$$

Here, $N_{Bu} = k\Delta$ is the Bouguer number, calculated for a characteristic length Δ , of the system.

The value of the integral $f_d(M, N_{Bu}, A_{\text{wall}})$ in Eq. (8), upon determination of the resolvent, is a function only of the coordinates of point M and of the optical-geometric properties of the radiating volume and surface.

The energy equation, on the basis Eqs. (1, 2, and 8), can be written:

$$4k(M)\sigma_0 [T^4(M) - T_{\text{wall}}^4] f_d(M, N_{Bu}, A_{\text{wall}}) + \gamma c \times$$

$$\left(w_x \frac{\partial T}{\partial x} + w_y \frac{\partial T}{\partial y} + w_z \frac{\partial T}{\partial z} \right) = 0 \quad (9)$$

Since we have assumed that the absorption coefficient of the medium is constant ($k = \alpha = \text{const}$), then for the case of one-dimensional flow of the medium along the x axis, Eq. (9) reduces to

$$4k\sigma_0 [T^4 - T_{\text{wall}}^4] f_d(M, N_{Bu}, A_{\text{wall}}) +$$

$$\bar{w}_1 \gamma_1 c f_w(y_1, z_1) dT/dx = 0 \quad (10)$$

where \bar{w}_1 and γ_1 are, respectively, the mean velocity and density of the medium in the initial cross section of the channel $f_w(y_1, z_1) = w_1(y_1, z_1)/\bar{w}_1$ is the normalized velocity distribution in the initial cross section of the channel ($x = 0$).

A solution of Eq. (10) for the case in which $f_d(M)$ does not depend on x , which is strictly valid for an infinitely long channel, is the following transcendental function of the temperature

$$16 \frac{k\sigma_0 T_{\text{wall}}^3 x f_d(M, N_{Bu}, A_{\text{wall}})}{\bar{w}_1 \gamma_1 c f_w(y_1, z_1)} =$$

$$\ln \left(\frac{T_{\text{wall}} - T_1}{T_{\text{wall}} + T_1} \frac{T_{\text{wall}} + T}{T_{\text{wall}} - T} \right) +$$

$$2 \arctan \left[\frac{T - T_{\text{wall}}}{T_{\text{wall}} + (T T_1 / T_{\text{wall}})} \right] \quad (11)$$

Equation (11)* can be written in the following dimensionless form:

$$16 \frac{f_d(M, N_{Bu}, A_{\text{wall}})}{f_w(y_1, z_1)} \frac{N_{Bu} x}{N_{B0} \Delta} = T^{\circ 3} \Phi(T^{\circ}, \theta)$$

where

$$\Phi(T^{\circ}, \theta) = \ln \frac{2/\theta + T^{\circ} - 1}{T^{\circ} + 1} +$$

$$2 \arctan \left[\frac{(\theta - 1)(T^{\circ} - 1)}{\theta T^{\circ}(T^{\circ} - 1) + T^{\circ} + 1} \right] \quad (12)$$

and

$$T^{\circ} = T_1 / T_{\text{wall}} \quad N_{B0} = \bar{w}_1 \gamma_1 c / \sigma_0 T_1^3$$

$$\theta = (T - T_{\text{wall}}) / (T_1 - T_{\text{wall}})$$

Here, N_{B0} is the Boltzmann number for the initial cross section of the channel; x/Δ is the dimensionless coordinate

* The second term on the right-hand side of Eq. (11) was misprinted in the original text—Reviewer.

along a streamline of the medium; θ is the dimensionless temperature at the point of the volume under consideration; T_1 and T_{wall} are, respectively, the temperatures in $^{\circ}\text{K}$ of the medium in the initial cross section of the channel and of the wall which bounds the channel.

For the case in which $T_{wall} = 0^{\circ}\text{K}$, the solution of the differential Eq. (10), represented in dimensionless form, is

$$\theta = (\pi_m)^{-1/3} \times \left(\pi_m = 1 + 12 \frac{x}{\Delta} \frac{N_{Bu} f_d(M, N_{Bu}, A_{wall})}{N_{Bo} f_w(y_1, z_1)} \right) \quad (13)$$

where π_m is a dimensionless parameter which depends on the entire set of parameters and on the coordinates of the point M under consideration.

Using Eq. (13), Eq. (12) can be rearranged as follows:

$$\pi_m = 1 + 0.75 T^{\circ 3} \Phi(T^{\circ}, \theta) \quad (14)$$

The curves in Fig. 1 show the dimensionless temperature field θ as a function of the parameter π_m and the temperature ratio $1/T^{\circ} = T_{wall}/T_1$, calculated from Eq. (14), which represents the general solution of the problem of radiant heat transfer in the one-dimensional flow of the medium.

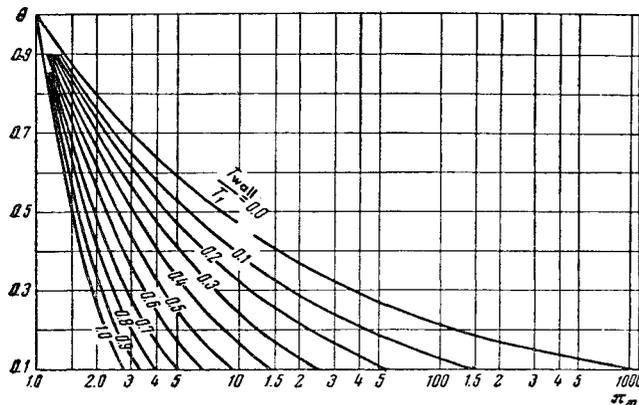


Fig. 1. Curves of dimensionless temperature in flow $\theta = (T - T_{wall}) / (T_1 - T_{wall})$ as a function of parameter π_m and of temperature ratio T_{wall}/T_1 , plotted from Eq. (14)

The dimensionless calorimetric temperature at the outlet cross section is determined by the expression

$$\bar{\theta} = \frac{1}{F_{BX}} \int_{(F_{BX})} \theta_2 \cdot f_w(y_1, z_1) dF_{BX} \quad (15)$$

where F_{BX} is the area of the outlet cross section of the flow, and θ_2 is the dimensionless temperature of the flow at the point of the outlet cross section under consideration.

The Bansen number, equal to the ratio of the temperature difference in the flow to the average temperature difference between the fluid and the walls is

$$N_{Ba} = \frac{T_1 - \bar{T}_2}{T_{flow} - T_{wall}} = 2 \frac{1 - \bar{\theta}}{1 + \bar{\theta}} \left(T_{flow} = \frac{T_1 + \bar{T}_2}{2} \right) \quad (16)$$

Here, T_{flow} is the average temperature of the flow, $\bar{\theta}$ is the dimensionless outlet calorimetric temperature defined in Eq. (15).

The coefficient of direct thermal efficiency σ , which represents the ratio of the amount of heat absorbed by the heated surface to that amount of heat which the flow would have emitted being cooled from its original temperature T_1 to the temperature of the heat absorbing surface T_{wall} , is equal (for $c = \text{const}$) to:

$$\sigma = 1 - \bar{\theta} \quad (17)$$

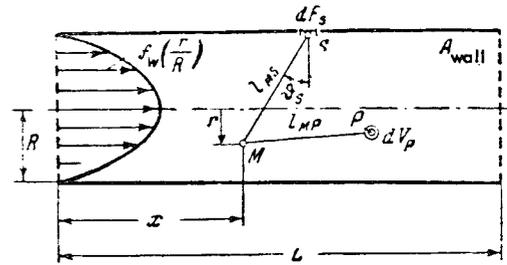


Fig. 2. Solution of problem of radiant heat transfer in a moving medium for a cylindrical channel

1. Solution of the Problem for a Cylindrical Channel

A purely absorbing medium with an absorption coefficient $k = \alpha$ (Fig. 2) is flowing through a cylindrical tube of diameter D . The emissivity of the wall of the cylindrical channel is $A(S) = A_{wall}$ and its temperature is $T(S) = T_{wall} = 0^{\circ}\text{K}$; both are constant over the entire surface. The temperature of the medium in the inlet cross section is also constant and equal to T_1 . The direction of the streamlines of the medium is considered parallel to the axis of the channel, whereas the velocity distribution over the inlet cross section is symmetric and is given by the distribution function

$$w_1/\bar{w}_1(r/R) = f_w(r/R)$$

where \bar{w}_1 is the average velocity in the inlet cross section of the flow.

The value of the integral in Eq. (8), on the basis of an analysis for cylinders with a large ratio of L/D , was obtained as follows

$$f_d \left(\frac{r}{R}, kD, A_{wall} \right) = \int_{(F)} \frac{\Gamma_{F-V}(M, S)}{4} A(S) dF_S = \frac{A_{wall} e^{-akR} ch(akr)}{A_{wall} + A_{med} - A_{wall}A_{med}} \quad (18)$$

where A_{wall} and A_{med} are the absorptivity of the wall and absorptivity of the cylindrical volume of the medium which fills the channel; $a \approx \sqrt{2}$ is an approximately constant coefficient for the cylindrical form of the channel; R is the radius of the cylindrical channel and r is the radial coordinate.

From the condition, $T_{wall} = 0^{\circ}\text{K}$, and utilizing Eq. (13), we obtain the following function for the dimensionless temperature in the cylindrical channel:

$$\theta \left(\frac{x}{D}, \frac{r}{R} \right) = \frac{T(x/D, r/R)}{T_1} = \psi(r) = \left(1 + 12 \frac{x}{D} \times \frac{N_{Bu} f_d(r/R, kD, A_{wall})}{N_{Bo} f_w(r/R)} \right)^{-1} \quad (19)$$

where $T(x/D, r/R)$ is the temperature in the point of the volume under consideration, which has the dimensionless coordinates x/D and r/R ; $N_{Bu} = kD$ is the Bouguer number and N_{Bo} is the Boltzmann number. As a characteristic dimension, the diameter of the cylindrical channel was selected.

The dimensionless calorimetric temperature of the flow in the outlet cross section is determined on the basis of (15) from the expression

$$\bar{\theta} = \frac{\bar{T}_2}{T_1} = \frac{1'}{\pi R^2} \int_0^R \frac{f_w(r/R) 2\pi r dr}{\psi(r)} \quad (20)$$

The Bansen number is calculated from (16) and (20). In analyzing Eqs. (20) and (16), it can be seen that the heat transfer as a function of the optical density of the medium is not at all a monotonous function as is the case in the previously indicated solutions, but passes through a maximum.

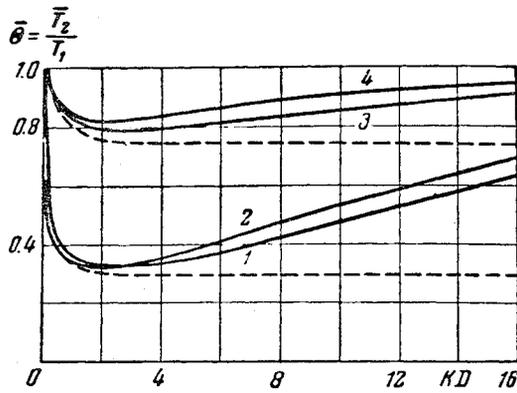


Fig. 3 Dimensionless calorimetric temperature at the outlet of $\theta = T_2/T_1$ as a function of Bouguer number $N_{Bu} = kD$ for a cylindrical channel: curves 1 and 3 are for a uniform velocity, curves 2 and 4 for a parabolic distribution [Eq. (21)]; curves 1 and 2 are for $N_{Bo} = 3$, curves 3 and 4 for $N_{Bo} = 80$; dotted curves—solutions (see Refs. 1-4)

For illustration of this fact, Figs. 3 and 4 show N_{Bo} as a function of the Bouguer number N_{Bu} , calculated from Eqs. (20) and (16). The calculations were made for two values of the Boltzmann numbers— $N_{Bo} = 3$ and $N_{Bo} = 80$ with a ratio of $L/D = 10$ and with uniform and parabolic distributions of velocity in the inlet cross section:

$$f_w(r/R) = 1 \quad f_w(r/R) = 2(1 - (r^2/R^2)) \quad (21)$$

The emissivity of the walls was taken equal to $A_{wall} = 1$. For comparison, these same curves show analogous functions calculated from the formula which was obtained by previous authors in Refs. 1-4 on the assumption of the constancy of the temperature in the transverse cross section of the channel.

In contrast to the generally accepted ideas, a characteristic peculiarity of the function obtained in this work is its extremal nature (passage of the heat transfer through a maximum with increasing optical density of the medium).

A noticeable influence on the radiant heat transfer in the flow is also exerted by the hydrodynamics which enter into the solution by the velocity distribution along the cross section of the channel. As is seen from the curves, with increasing non-uniformity of the velocity distribution, the heat transfer decreases, the more noticeably, the greater the Boltzmann number.

The resulting maximum, although it seems on first view as unexpected, is actually well confirmed by the physical ideas, experimental data, and certain special analytical investigations of radiant heat transfer (see Refs. 6 and 7).

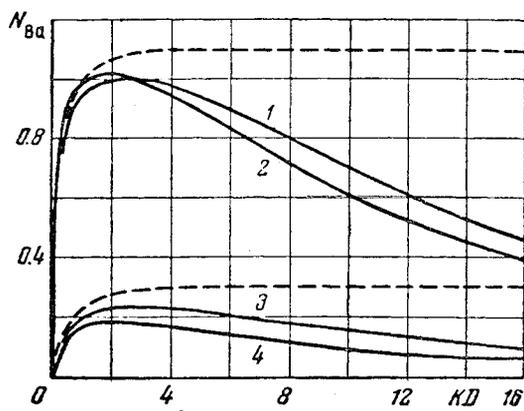


Fig. 4 Bansen number N_{Ba} as a function of Bouguer number N_{Bu} for a cylindrical channel: curves 1 and 3 are for a uniform distribution of velocity, curves 2 and 4 for parabolic distribution [Eq. (21)]; curves 1 and 2 are for $N_{Bo} = 3$, curves 3 and 4 for $N_{Bo} = 80$; dotted lines—solutions (see Refs. 1-4)

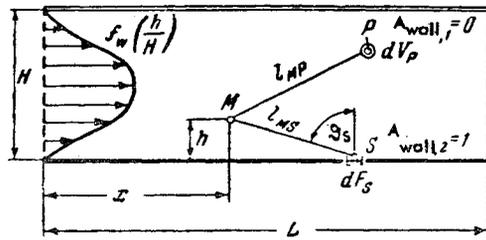


Fig. 5 Solution of the problem of radiant heat transfer in a moving medium for a plane channel

The resulting extremal nature of the function from a physical point of view is determined by the screening action of the cold layers of the gas, which are situated closer to the wall of the channel. The effect of decreasing the emissivity of the medium with an increase of its optical density to a certain extent is contained also in the purely empirical formulas which determine the radiation heat transfer in the moving medium. For example, in comparing the norms of the calculation of the boiler units, this effect was considered by the introduction of a table of correction coefficients which depend on the type of fuel and which decrease for optically denser media.⁸

2. Solution of the Problem for a Plane Channel

A "gray" medium with a temperature in the initial cross section T_1 , average speed \bar{w}_1 , and absorption coefficient $k = \alpha$ moves into an infinite plane channel with a height of H . The upper wall of the channel has an emissivity $A_1 = 0$ and the lower wall, $A_2 = 1$ (Fig. 5). The velocity distribution with respect to the initial cross section of the channel is given by the function

$$w_1/\bar{w}_1(h/H) = f_w(h/H)$$

The flow of the medium is one-dimensional just as for the cylindrical channel. The streamlines are parallel to the lateral planes of the channel. The temperature of the lower radiation-receiving wall is equal to T_{wall} .

The value of the integral in Eq. (8) for a sufficiently long channel is obtained analytically as follows:

$$f_a\left(\frac{h}{H}, kH\right) = \int_{(F)} \frac{\Gamma_{F-v}(M, S)}{4} A(S) dF_S = e^{-ak0.5H} \times \left[\cosh(akh) - \frac{A_{med}e^{-akh}}{2} \right] \quad (22)$$

where $A(S) = A_{wall} = 1$ is the absorptivity of the radiation-receiving surface F_2 , A_{med} is the absorptivity of the plane layer of the medium which fills the channel, with a height H and absorption coefficient k , determinable from known formulas, h is the vertical coordinate, $a \approx 2.83$ is an approximately constant coefficient for a plane layer.

The curves in Fig. 6 show the function $f_a(h/H, N_{Bu})$ for a plane layer with $A_1 = 0$ and $A_2 = 1$, which will henceforth be utilized in the calculations.

Since the temperature of the heat-accepting surface of the plane channel differs from zero, $T_{wall} \neq 0$, then for the determination of the temperature field, it is necessary to use Eq. (13). For this case, the parameter π_m is given by the formula

$$\pi_m = 1 + 12 \frac{N_{Bu} x}{N_{Bo} H} \frac{f_a(h/H, N_{Bu})}{f_w(h/H)}$$

in which $N_{Bu} = kH$ is the Bouguer number, N_{Bo} is the Boltzmann number, and x/H and h/H are dimensionless coordinates of the point under consideration.

As a characteristic length for this case, the channel height H was selected. By using the resulting function, it is possible

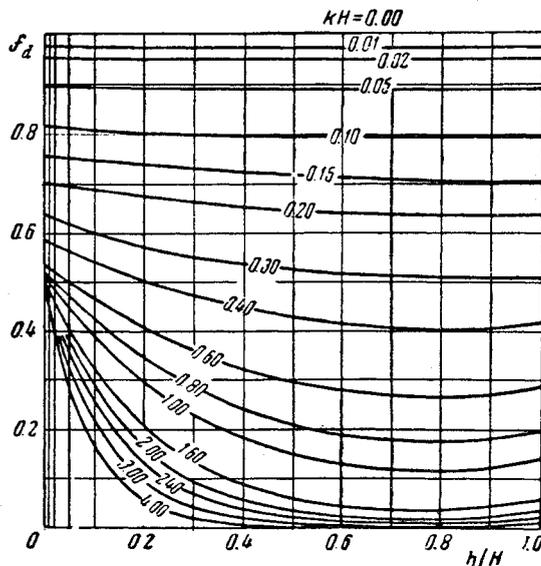


Fig. 6 Curves of $f_d(h/H, N_{Bu})$ as a function of dimensionless height h/H and of Bouguer number $N_{Bu} = kH$ for a plane layer with $A_2 = 1$ and $A_1 = 0$

to calculate the temperature field in the flow and the heat transfer characteristics for a plane channel.

The curves in Fig. 7 show the coefficients of direct thermal efficiency, calculated from Eqs. (14, 15, and 17), applicable to actual magnitudes of values encountered in metallurgical furnaces ($N_{Bo} = 2.20$, $L/H = 3.60$, $T_{wall}/T_1 = 0.70$) as a function of the Bouguer number $N_{Bu} = kH$ for different hydrodynamic characteristics of the flow. The distribution of velocity in the inlet cross section, which represents the hydrodynamic characteristics of the flow, was investigated for three profiles (diagrams 1, 2, and 3 in Fig. 7), expressed by the functions

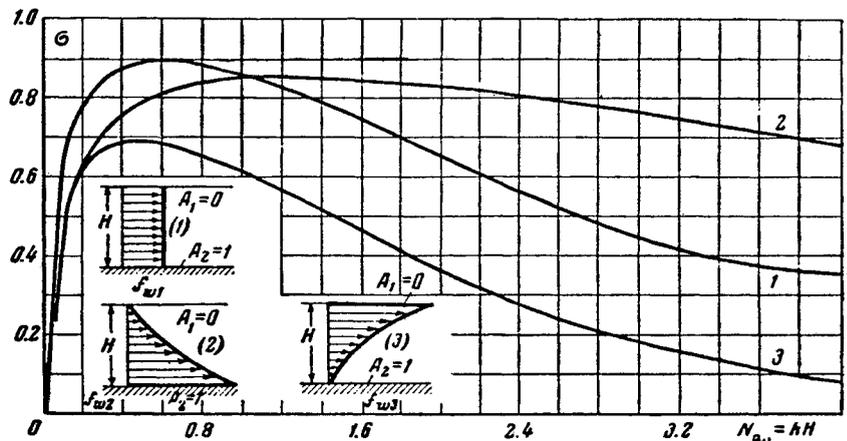
$$f_{w,1}(h/H) = 1 \quad f_{w,2}(h/H) = 3[1 - (h/H)^2]$$

$$f_{w,3}(h/H) = 3(h/H)^2$$

First of all, it becomes obvious from the curves in Fig. 7 that there is a maximum of heat transfer as a function of the optical density of the medium just as for a cylindrical channel. With increasing optical density, the heat transfer increases up to a definite maximum, after which, with further increase of optical density, the heat transfer gradually decreases to zero. The shape of the curve and the position of its maximum are thereby found to be strongly dependent on the hydrodynamic characteristics of the flow.

It is obvious from the curve that the uniform velocity distribution gives a greater coefficient of direct thermal efficiency in comparison with the flat movement of the flow along the radiation-receiving F_2 in accordance with diagram only as

Fig. 7 Coefficients of direct thermal efficiency as a function of Bouguer number N_{Bu} in a plane channel for different velocity distributions in the inlet cross section



long as the values of the Bouguer number are of the order of 1. The flat movement along the radiation-receiving surface F_2 (diagram 2) gives the same effect of heat transfer as the flat movement along the reflecting surface F_1 (diagram 3) as far as the values $N_{Bu} = kH = 0.15$; further, the curves diverge sharply and the advantage remains with diagram 2. The influence of the optical density of the medium and of the hydrodynamics of the flow on the radiant heat transfer turns out to be similar for both the plane and cylindrical canal.

—Submitted February 18, 1958

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Reviewer's Comment

The mathematics employed in this paper is more elegant than in the authors' previous treatment of this subject [see "Heat transfer in the channel flow of radiating combustion products," *Teplenerg.* (Thermal Power), no. 3, 50 (1957)], but the results obtained are identical. In their earlier paper, the energy equation was written

$$4k\sigma_0 T^4 f_a + \rho u C_p (dT/dx) = 0$$

where $f_a = (\frac{1}{4}\pi) \int_{4\pi} \exp(-kl) d\omega$ is the transmission factor, k

the (constant) linear absorption coefficient, l the distance between the volume element of the gas and the surface of the channel, and ω the solid angle. They stated that the trans-

mission factor had been evaluated analytically for a long cylindrical channel, with the result

$$f_a = \exp(-\sqrt{2} kR) \cosh(\sqrt{2} kr)$$

but gave no reference to where this was done.

The authors' solution, which neglects thermal conduction and heat generated by viscous dissipation, is presented in terms of the Bouguer number, the Boltzmann number, and the Bansen number. These dimensionless numbers are defined as follows.

The Bouguer number N_{Bu} is equal to the product of the linear absorption coefficient of the gas k (which, in general, is a function of the temperature and pressure of the gas, as well as of the wavelength of the radiation), and of some characteristic length for the geometry considered, and would correspond to what astrophysicists term the "optical depth," if the characteristic length were chosen along the direction of the radiant flux vector.

The Bansen number $N_{Ba} = (T_1 - \bar{T}_2)/(T_{flow} - T_{wall}) = (h_{rad}F)/(mC_p)$, where F is the surface area of channel walls, $m = \rho uA$ the mass flow rate of the medium through a channel of cross-sectional area A , \bar{T}_2 the "cup-mixing" temperature at the exit section, $T_{flow} = (T_1 + \bar{T}_2)/2$ is the mean temperature of the flow, T_{wall} the temperature of the channel wall, and h_{rad} the radiative heat transfer coefficient per unit area of the surface defined by $q_r = h_{rad}F(T_{flow} - T_{wall})$. The Bansen number is analogous to the Stanton number $N_{ST} = (h_{conv}/\rho u C_p)$ for convective heat transfer where h_{conv} is defined by $q_c = h_{conv}A(T_{flow} - T_{wall})$ and, in fact, $N_{Ba} = N_{St}(h_{rad}F)/(h_{conv}A)$.

The Boltzmann number $N_{Bo} = (\rho_1 u_1 C_{p1}/\sigma_0 T_1^3)$, where the subscripts 1 refer to the entrance section of the channel. This is obviously closely related to the Bansen number, since $(N_{Ba})(N_{Bo}) = (h_{rad}F)/(A\sigma_0 T^3)$.

By assuming the walls of the channel to be totally black and at 0°K, radiant energy exchange between the various surface elements of the channel would be neglected. The simplified problems treated by Adrianov and Shorin are useful in giving insight into how radiation can influence the flow field, the temperature distribution, and the total heat transfer rate to the channel walls. The energy equation with radiation included is extremely complicated, since it is a nonlinear, integrodifferential equation. Some recent approximate solutions have been obtained by other authors essentially by the application of existing emissivity information to flow and temperature distributions calculated on the basis of a non-radiating medium.¹⁻¹¹ Also, several estimates of the coupling between radiative and convective transport in detached shock layers have been made.¹²⁻¹⁴ The problem of combined convective and radiative energy transfer has been treated for the limiting cases of an optically thin medium,¹⁵⁻¹⁷ an optically dense medium,¹⁸ and a medium completely transparent to radiation.¹⁹⁻²² The problem of combined conductive and radiative energy transfer has been treated for an optically dense medium,²³⁻³¹ where the diffusion approximation is applicable. To the reviewer's knowledge, no papers have appeared in which the combined effects of radiation, conduction and convection are treated analytically.

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